Bilinear or linear fractional or Mobius Transformation

(6h)

Definition: A transformation T defined by

$$w = T(z) = \frac{az+b}{cz+d} \quad \text{--- (1)}$$

where a, b, c, d are complex numbers and $ad-bc \neq 0$ is called a bilinear transformation or a linear fractional or Mobius transformation.

The restriction $ad-bc \neq 0$ is necessary because if $ad-bc=0$, then $a/c = b/d$ (say) implies that $w = T(z) = k$ = a constant independent of z. Thus the entire z-plane is mapped into the same point in the w-plane.

Here ad-bc is called the determinant of the transformation. [redacted] If $ad-bc=1$, the transformation (1) is said to be normalised.

The transformation (1) may be written as

$$cwz + dw - az - b = 0 \quad \text{--- (2)}$$

Evidently (2) is linear in both z and w. Hence it is called a bilinear transformation.

It is also called a linear fractional transformation or a Mobius transformation after the name of A.F. Mobius (1790 — 78/18..

A.F. Mobius was a German mathematician who was one of the best known of Gauss's pupils. A.F. Mobius published Barycentrische calcul in 1826. Mobius Collected works were published at Leipzig in four volumes (1885—1887).

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~~Extended Complex Plane~~ Critical points

Consider the bilinear transformation

$$w = T(z) = \frac{az+b}{cz+d}$$

Solving this for z, we get the inverse map as

$$z = T^{-1}(w) = \frac{-dw+b}{cw-a}$$

Transformation T associates each point of z-plane [redacted] a unique point of w-plane except the point $z=-\frac{d}{c}$ when $c \neq 0$. The transformation T^{-1} ~~associal~~ associates each point of w-plane with a unique point of z-plane except the point $w=\frac{a}{c}$, when $c \neq 0$. These exceptional points

$z = -\frac{d}{c}$ and $w = \frac{a}{c}$ are mapped into the points $w = \infty$ and $z = \infty$

It is clear that $\frac{dw}{dz} = \frac{(cz+d)a - (az+b)c}{(cz+d)^2} = \frac{ad-bc}{(cz+d)^2}$

$$\Rightarrow \frac{dw}{dz} = \begin{cases} \infty & \text{if } z = -\frac{d}{c} \\ 0 & \text{if } z = \infty \end{cases}$$

These points $z = -\frac{d}{c}$ and $z = \infty$ are called critical points where the conformal property does not hold good.

If the complex plane is closed by addition of the point ∞, then we say that the bilinear transformation sets up one-one correspondence between all points of the closed z-plane and the closed w-plane.

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